



2.1.3 Geometric interpretation of real 2- and 3-dimensional vectors





Introduction

- In this topic, we will
 - Look at geometric interpretations of 2-dimensional vectors
 - Consider geometric interpretations of 3-dimensional vectors



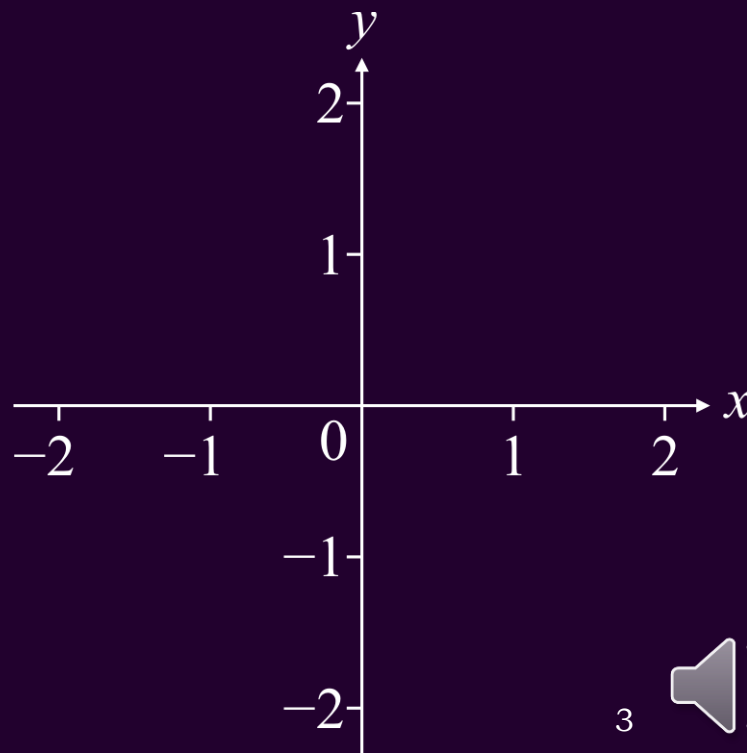


2-dimensional vectors

- Real two-dimensional vectors can be interpreted as points on a plane
 - Generally, the axes are labeled x and y
 - 2-d vectors are interpreted as coordinates in the x - y -plane

$$\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$$

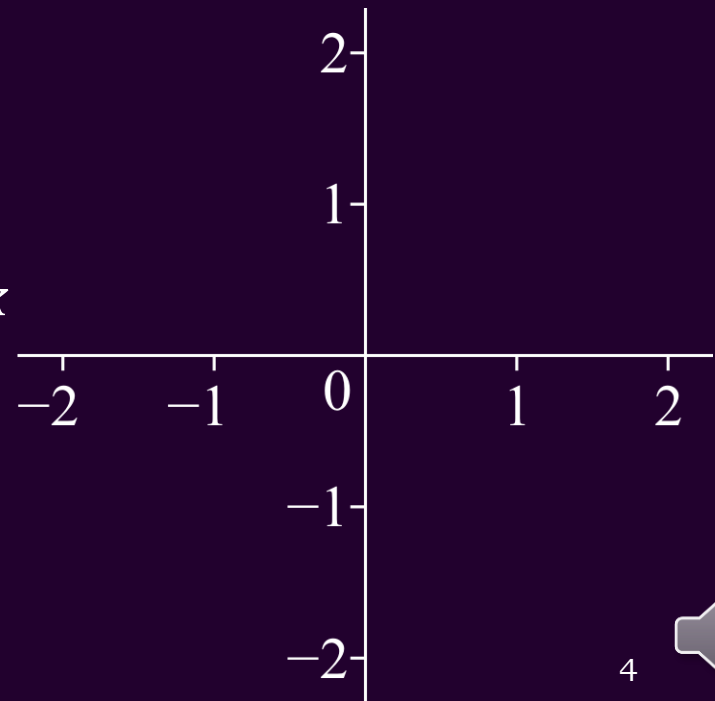
- The first entry corresponds to the x coordinate
- The second entry corresponds to the y coordinate





2-dimensional vectors

- We may or may not
 - Put labels on the axes
 - Put arrow heads on the axes
- You should understand the significance of the positions
 - Unnecessary *ink* on a graphic that does not help comprehension is referred to by Tufte as *chartjunk*
- In some cases, if the actual values don't matter, we may not even include the tick marks and labels





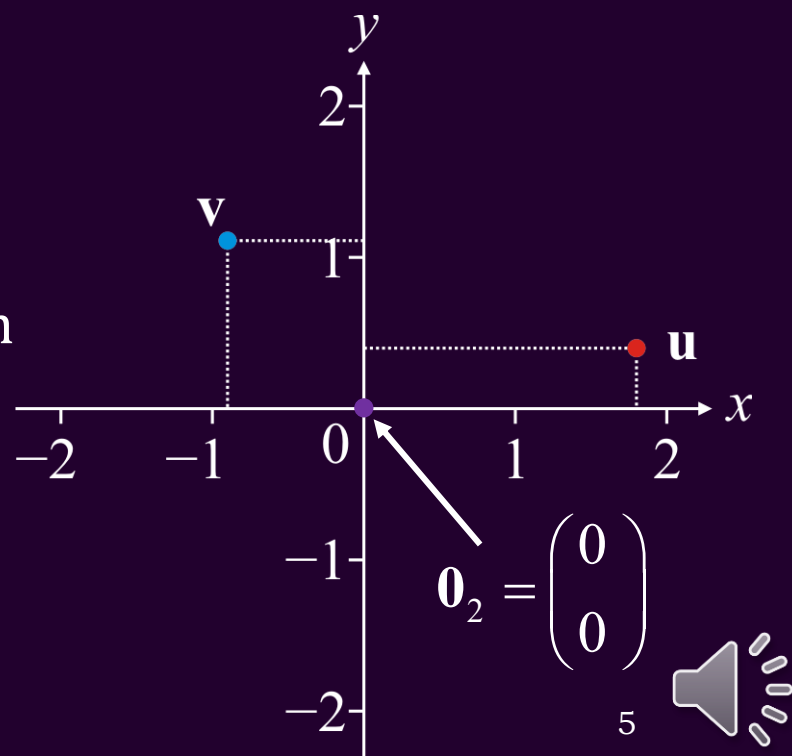
2-dimensional vectors

- Here are two examples

$$\mathbf{u} = \begin{pmatrix} 1.8 \\ 0.4 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} -0.9 \\ 1.1 \end{pmatrix}$$

- The zero vector is at the intersection of the axes



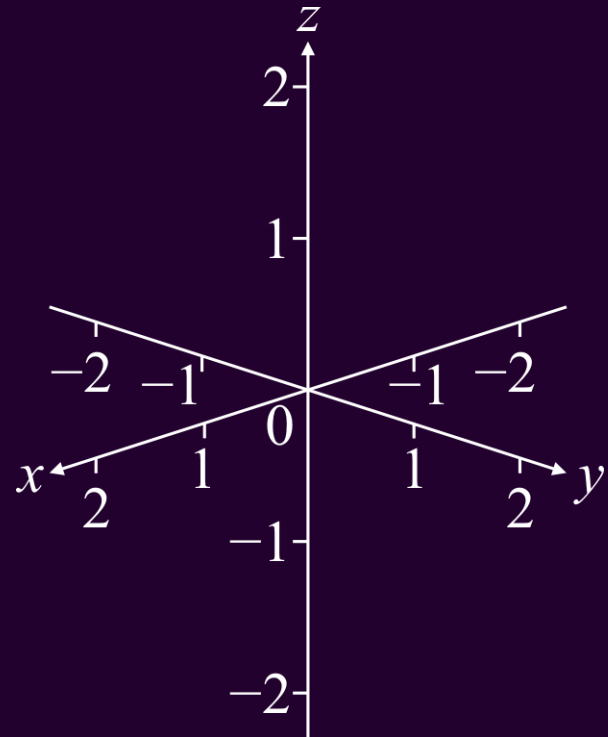


3-dimensional vectors

- Real three-dimensional vectors can be interpreted as points in space
 - Generally, the axes are labeled x , y and z
 - 3-d vectors are interpreted as coordinates in 3-space

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

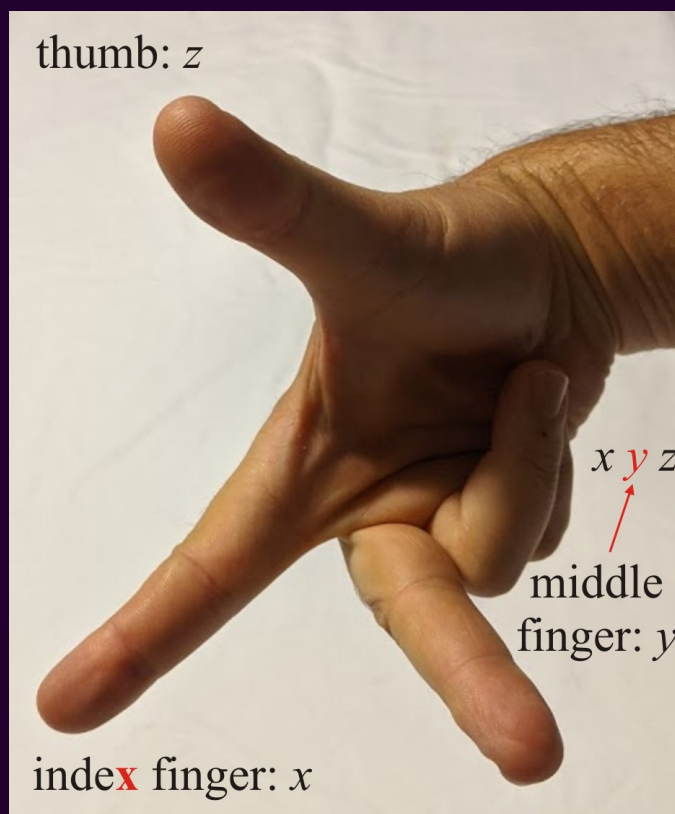
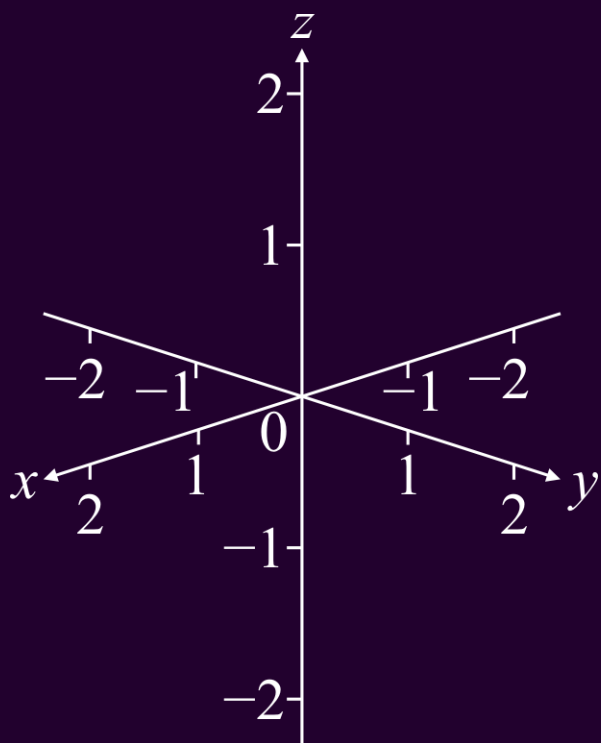
- The orientation of the positive x -, y - and z -axes depends on the *right-hand rule*





3-dimensional vectors

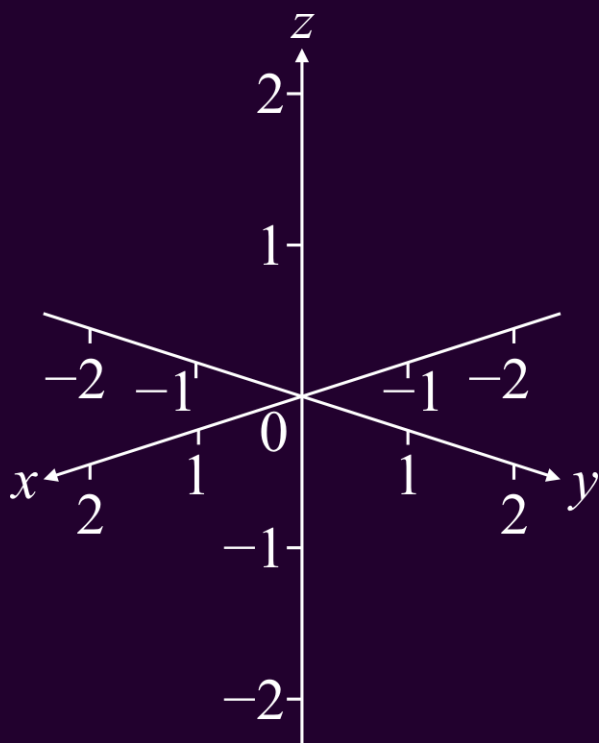
- The right-hand rule indicates the directions of the positive x -, y - and z -axes using the index and middle fingers and the thumb:





3-dimensional vectors

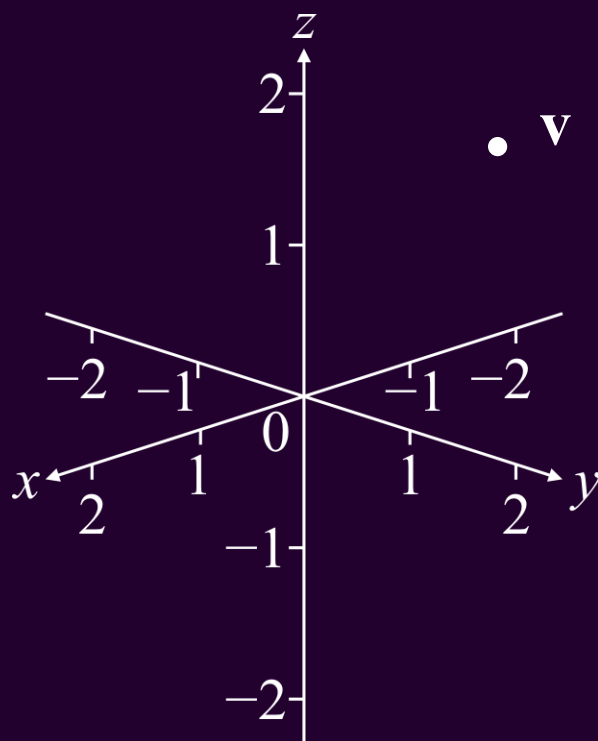
- Alternatively, if the thumb points in the direction of positive z , and the fingers point in the direction of positive x , curling the fingers moves towards positive y





3-dimensional vectors

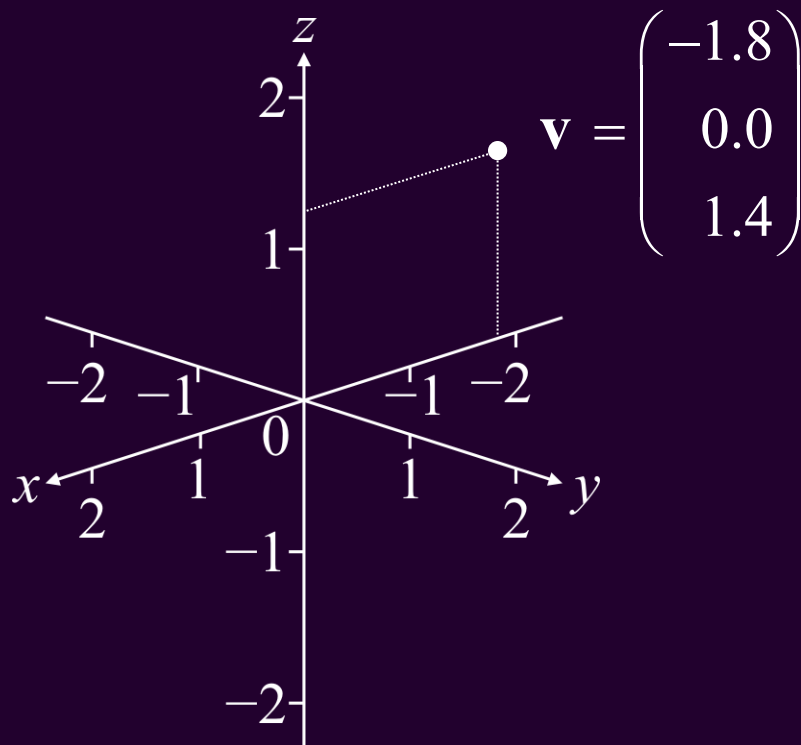
- Visualizing a point in 3-space can be more challenging
 - It requires visual cues





3-dimensional vectors

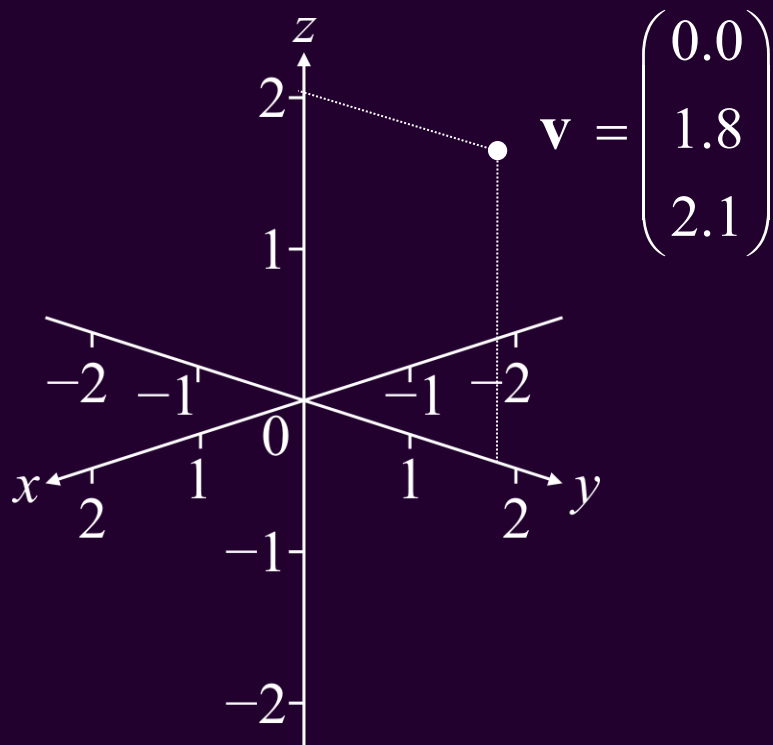
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3-dimensional vectors

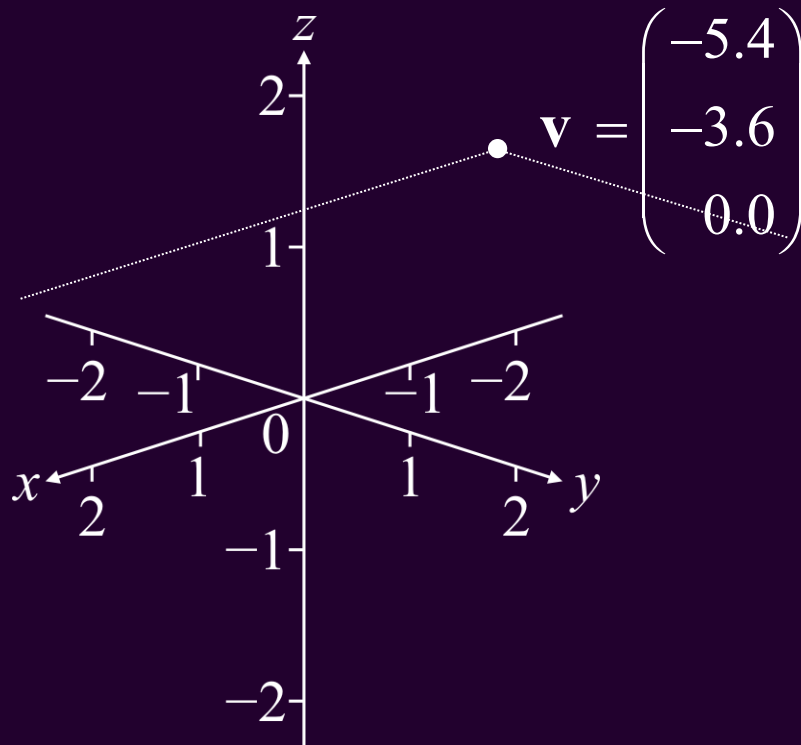
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3-dimensional vectors

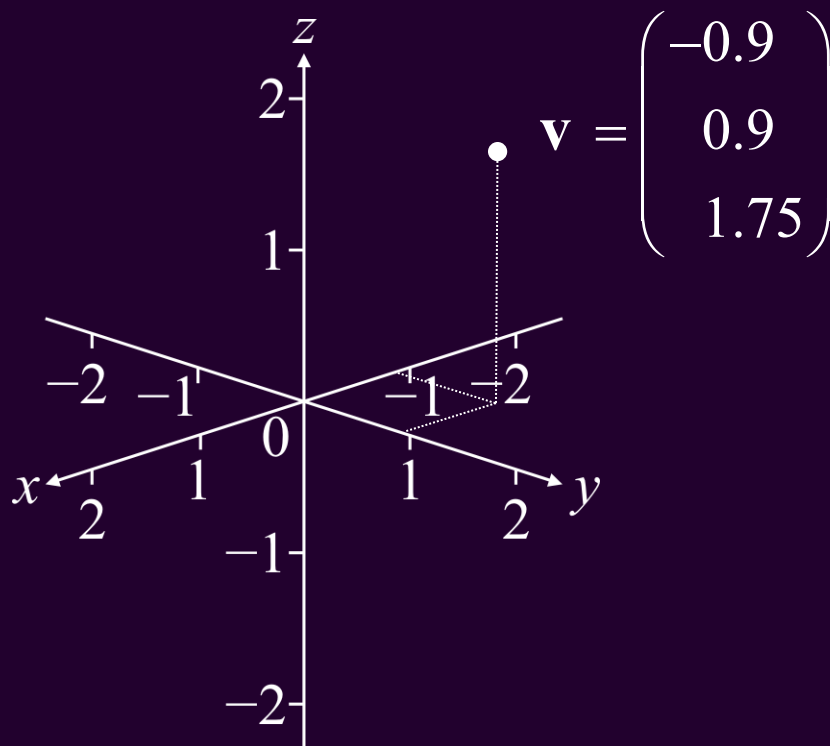
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3-dimensional vectors

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Summary

- Following this topic, you now
 - Seen interpretations of 2-dimensional vectors
 - Seen interpretations of 3-dimensional vectors
 - Are aware of the right-hand rule for the axes in 3-space





References

- [1] https://en.wikipedia.org/wiki/Linear_algebra





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

<https://www.rbg.ca/>

for more information.





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